

Mystery of Plane Symmetry

A Philosophical Approach to Mirror Image and Shadow

1 Introduction

E.T.A. Hoffmann's "A New Year's Eve Adventure" (1815) is the story of a man who has sold his mirror image to a demonic woman. This work is not only an homage to Albert von Chamisso's "The Curious Tale of Peter Schlemihl" (1814), but also features Peter Schlemihl, the man who sold his shadow to the devil. Both men are ostracized by the secular world and set out on a meandering journey in search of lost mirror images or shadows. There is no shortage of literary works whose subject is mirror image or shadow, and such works are often linked to self-identity. It has been pointed out that paintings have their origins in mirror images and shadows (Okada 2010, 17). Seen from this perspective, they can be interpreted as the inception of human desire to copy and preserve the identity of objects. René Magritte's "Not to Be Reproduced" (1937) might depict a mirror reflecting his ironic criticism against such a tendency (see Fig. 1).

If we could see our own face directly, what would be the difference between that face and the one reflected in the mirror? When we face a plane mirror placed vertically on the ground, the mirror image is horizontally reversed. If we raise our right hand, the mirror image will raise its left hand. However, this "left-right reversal" is a misleading expression. For example, when you are driving a car and the car behind tries to overtake you from the right, the car you see in your rearview mirror gradually disappears to the right. In this case, we do not think that the following car is moving to the left. In other words, our sense of left-right reversal is lost. Suppose the vehicle behind is an ambulance. In the rearview mirror, you will see the word "ambulance" painted on its hood. If you look directly behind you, you will see that the letters are reversed left and right (see Fig. 2). Thus, the mirror image is indeed reversed, but there is a problem that cannot be explained by geometrical optics. The correct answer to mirror reversal is quite simple: "Sign

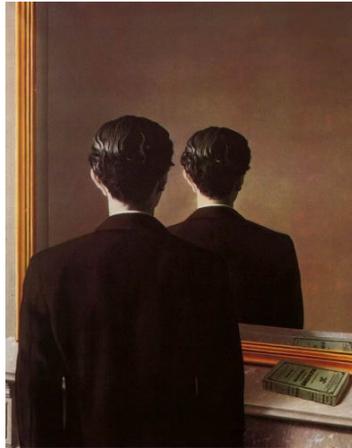


Figure 1. Magritte, René, *Not to Be Reproduced*, 1937, Museum Boijmans Van Beuningen, Rotterdam, <https://www.renemagritte.org/not-to-be-reproduced.jsp>.

AMBULANCE

Figure 2. Mirror Writing (AMBULANCE).

inversion of the axis perpendicular to the mirror surface” or “Plane symmetric transformation on the mirror surface as the symmetry plane” (Yoshimura 2004, 10). Nevertheless, mirror reversal is known to be a controversial topic. This is partly because it allows room for psychological and philosophical interventions beyond the geometrical answers mentioned above, but I suspect that one of the main reasons is that it has been involved in a kind of interdisciplinary struggle before that.

What follows can be separated into two parts. First, I critically examine the theories of several researchers who have tackled the mystery of mirror reversal, and make remarks on applying frame of reference (from the intrinsic frame of reference and the common frame of reference) to mirror image. Second, I propose a hypothesis that mirror image and shadow have a commonality in applying frame of reference. Throughout this chapter, I analyse the plane symmetry of them and its attribution to the self.

2 Mirror image

2.1 *Multiple frames of reference*

Mirror reversal is a topic that has long attracted attention. Indeed, it is mentioned in Plato's "Timaeus", but here I will first quote an explanation by Richard Feynman, a famous theoretical physicist:

We think of our image as another person. We cannot imagine ourselves 'squashed' back to front, so we imagine ourselves turned left and right, as if we had walked around a pane of glass to face the other way. It is in this psychological turnabout that left and right are switched. (Gleick 1992, 332)

To rephrase Feynman's explanation with some interpretation, the left-right reversal of the mirror image is caused by the difference between the mirror image and internal representation of our own body when we mentally rotate it 180° around an axis parallel to the mirror surface. At the same time, Feynman pointed out that the orientation other than the axis perpendicular to the mirror surface does not change (Ibid., 332). For example, if we raise our right hand in front of a mirror and it points to the west, the left hand of the mirror image also points to the west. In contrast, the direction of our face would be reversed. From this, we see that left and right in the subjective frame of reference are different from east and west in the objective frame of reference – orientation. This may be one of the reasons why the mirror image inversion problem is so complicated. In other words, we usually choose the internal frame of reference centered on the self, the external frame of reference centered on the environment, and the orientation, which is a more global and objective frame of reference, without being particularly conscious of them, but the mirror image – especially the mirror image of one's own body – is special, and it is possible to presume what can be called the internal frame of reference from the mirror image viewpoint. Therefore, multiple frames of reference are expected to appear in the explanation of mirror reversal. In the following, I will discuss the theories of a psychologist and a physicist who are at the centre of the controversy, and give consideration to the complicated problem of mirror reversal.

2.2 *Multi-process theory*

According to psychologist Yotaro Takano, mirror-image reversal can be explained by optic reversal, representational reversal, and viewpoint reversal, all of which are phenomena produced by any of the three processes of optic



Figure 3. Examples of Representational Reversal. If you are familiar with Japanese Kanji characters, you will recognize the letter on the left side is reversed.

transformation, representational transformation, and viewpoint transformation, or a combination thereof (Takano 1998, 2015). Takano calls it multi-process theory, and he asserts that the mirror reversal when an observer looks at his or her own body in a mirror and the mirror reversal of letters are the products of different processes. The following details the specifics of the multi-process theory.

(i) Optic reversal

Optic reversal refers to the reversal in the direction perpendicular to the mirror surface. For example, when the mirror surface and the real object are facing each other, the depth – the front and back – of the mirror image and the real object will be reversed. This optic reversal is based on the basic laws of geometrical optics (the angle of incidence is equal to the angle of reflection, and straightness of light). The above-mentioned process that causes reversal is optical transformation.

(ii) Representational reversal

Representation reversal refers to the reversal that occurs when the shape of an object in memory is different from its mirror image. For example, when an asymmetrical character in the language used by the observer is placed face to face with a mirror, the mirror reversal can be seen without checking the actual character (see Fig. 3). The above process of comparing memory and mirror image is representational transformation.

(iii) Viewpoint reversal

Viewpoint reversal refers to the reversal of the frame of reference to the mirror-image viewpoint, and its tendency is particularly evident with the mirror reversal of the observer's own body. Takano suggests that mental rotation as described by Feynman is not necessarily required, and that when left and right are determined from the observer's viewpoint, the left and right of the

mirror-image viewpoint can also be determined immediately. The process of reversing the frame of reference in this way is called viewpoint transformation.

According to Takano's explanation, frame of reference is reversed only in (iii), and the frame of reference centered on the observer is used in (i) and (ii). In addition, optic transformation is required for mirror reversal in ii) and iii). This rather complicated and confusing description is a characteristic of multi-process theory, which mixes up the issues of "geometrical-optical left-right reversal" with "presence/absence of left-right reversal recognition," which strictly speaking should be distinct. For example, if we see only the mirror image of mirror writing, we may not feel any sense of left-right reversal if the letters are familiar to us; or, if the letters are unfamiliar, then we may not be able to determine whether the letter is reversed or not. In both cases, however, geometrical-optical left-right reversal is occurring.

2.3 Geometrical-optical approach

According to the physicist Tatsuo Tabata, "left and right" are properties that depend on "up and down" or "front and back" in three-dimensional space (Tabata and Okuda 2000). This means that if either the up-down axis or the front-back axis is reversed, the left-right axis will be reversed subordinately. For example, when an observer faces a mirror, A) the up-down axis of the real object and the mirror image remain unchanged, and B) the front-back axis is reversed. In this case, the left-right reversal is derived a posteriori from the facts A) and B). Similarly, if the observer stands on a mirror placed horizontally on the floor, C) the up-down axis of the real object and the mirror image will be reversed, and D) the front-back axis will remain unchanged. In this case, the left-right reversal also occurs in C) and D). The geometrical-optical approach appears to be quite clear, but there is a blind spot. It can be seen in the case where the object is facing sideways to the mirror surface. In this case, the up-down and front-back axis of the object and the mirror image do not change, but the left-right axis is reversed. In geometrical-optical approach, the case where the axis perpendicular to the mirror surface coincides with the left-right direction is treated as an exception. This is also the case in the multi-process theory, where the explanation is based on a switch from viewpoint reversal to optic reversal (Takano 2015, 124).

Tabata's theory does not include psychological processes. It is a genuinely geometrical-optical explanation of mirror reversal. The question that arises in response to this explanation is, "Why do the up-down and front-back axis take precedence over the left-right axis"? Tabata asserts that the reason is that many

people and objects are left-right symmetrical. In other words, the left-right direction is decided subordinately because, generally, left and right are similar, while there are clear differences up and down, or front and back. I will not go into more detailed criticism in this chapter, but suffice to say that it is presumed that there is no case in which the up-down axis and the front-back axis are both reversed in the relationship between the real object and its mirror image. Furthermore, as I mentioned in the previous section, mirror reversal is “reversing the axis perpendicular to the mirror surface.” Therefore, the reason why the up-down axis and the front-back axis are given priority can be restated as follows:

When the observer or the object and the mirror are placed face to face, the front-back direction, which coincides with the axis perpendicular to the mirror surface, is reversed first. Similarly, when the observer or the object is on a mirror placed horizontally on the floor, the up-down direction, which coincides with the axis perpendicular to the mirror surface, is reversed first.

This means that, with the exception of the case where the observer or the object faces sideways to the mirror surface – the left-right direction coincides with the axes perpendicular to the mirror surface – it is still true that the reversal of the front-back and up-down axis takes precedence over the left-right axis. Furthermore, when the left-right axis reverses, the front-back and up-down axis do not reverse along with it, providing evidence to support the priority of the front-back and up-down axis. However, since the priority of the front-back and up-down axis is based on the asymmetry of the observer or the object, the fact that the front-back and up-down axis are generally indistinguishable does not apply to spheres.

If there are phenomena that cannot be covered by geometrical-optical approach, what are they? According to Tabata, “psychology is concerned with the motivation for choosing the intrinsic frame of reference or common frame of reference, the mechanism of selection and conversion of them, and the cognitive process in the selected frame of reference” (Tabata 2008, 515). Here, the intrinsic frame of reference refers to one that is applied separately to the observer or the object and the mirror image, while the common frame of reference refers to one that is applied to the observer or the object and the mirror image. As mentioned at the beginning of this chapter, the selection of frame of reference is something that we do in our daily lives automatically, without thinking about it. However, the choice of frame of reference is one of the factors that complicate the mirror image problem, namely, the presence or absence of the sense of left-right reversal.

2.4 Focusing on controversial points

Comparing the theories discussed in [Section 2.2](#) and [2.3](#), a discrepancy becomes apparent in their definitions of “mirror reversal.” In particular, there is a major difference in how the mirror reversal of letters is explained. According to the multi-process theory, mirror reversal of letters can be confirmed by optic reversal or representational reversal. Since optic reversal is based on geometrical optics, it is not considered to be the cause of discrepancies with other theories – given that it is obvious that there is no theory that denies geometrical-optical facts. However, there is a dissonance of opinion among theorists as to whether or not representational reversal can be included in mirror reversal. The reason is that representational reversal can be regarded as the recognition of differences through pattern matching. Representational transformation refers to the process of comparing the shape of a character in memory with a mirror image, but even if it is mirror writing on paper, the sense of left-right reversal can still occur. Since we usually do not have a sense of left-right reversal to the mirror image of mirror writing, we cannot apply the representational transformation to the mirror image. In other words, the representational transformation explains the process of causing the sense of left-right reversal beyond the simple geometrical-optical reversal, and it is questionable to treat representational reversal as mirror reversal.

It is assumed that we do not usually take the viewpoint of the letter. This means that we do not generally apply the viewpoint transformation of multi-process theory, or the intrinsic frame of reference of geometrical-optical approach to letters. According to Thomas Nagel, we cannot have the phenomenal experience of being a bat (Nagel 1974), but we cannot even begin to imagine what it means to be a letter. It can be assumed that we usually do not have a sense of left-right reversal to mirror image of objects, but letters are exceptions. Therefore, it is possible to understand the intention to treat letters as a special case of mirror reversal. From a geometrical-optical point of view, however, there is no difference between the case of the observer and that of other objects.

2.5 Applying a frame of reference

We can apply frames of reference to more than just the mirror image in our everyday lives. For example, suppose we have a cup and a plate on a table. Then we can assume the internal frame of reference – an absolute positional relationship between them and us, and the external frame of reference – is a relative positional relationship between the items. However, the expression “a

cup is on the right side of a plate” clearly relies on the viewpoint of a specific observer, albeit a relative positional relationship. Therefore, in everyday life, the external frame of reference that is applied to objects, such as dishes, with generally unspecified front-back and left-right positions, is often a copy of the internal frame of reference. It is clear from the previous discussion that the mirror image has a special meaning when choosing a frame of reference. This is because we can adopt the viewpoint of a mirror image using viewpoint transformation in multi-process theory, or choosing the intrinsic frame of reference in the geometrical-optical approach. This process is different from the usual choice of internal frame of reference or external frame of reference. In other words, it is like transferring ourselves to others’ point of view.

If we use the terms “the intrinsic frame of reference” and “the common frame of reference” in accordance with the geometrical-optical approach, the choice of these frames of reference causes the mirror image to be reversed, or not. Are there objects, then, to which we tend to apply the intrinsic frame of reference to their mirror images? James Gibson, an ecological psychologist, uses the terms “detached object” and “attached object” in his classification of the things that make up the environment (Gibson 1979). Simply put, a detached object is an object that moves, a person being a typical example. On the other hand, an attached object is an object connected to the ground in most cases, such as a house or a tree. It is assumed that we tend to apply the intrinsic frame of reference to detached objects, especially those that are asymmetrical and can be defined in regard to their up and down, front and back. Therefore, it is relatively easy to apply the intrinsic frame of reference not only to people but also to animals and vehicles. In other words, we can draw a boundary between objects whether or not we can take the viewpoint of the object to which we are transferring.

I would now like to return to the highway referred to in the Introduction. What you see in the rearview mirror are, roughly speaking, the ambulance behind, the letters its hood, and the driver’s face. If the driver of your car is you, you must apply the common frame of reference to all the objects. Otherwise, we might recognize the direction of the ambulance in the opposite. Even when we face a mirror in our everyday lives, we are barely aware of the fact that the left and right sides are reversed. Ultimately, therefore, the application of the intrinsic frame of reference to mirror images is extremely limited, and the common frame of reference is chosen in most cases. Why is that?

3 Shadow

“There is strong shadow where there is much light,” is an impressive phrase that appears in Goethe’s play “Götz von Berlichingen with the Iron Hand” (1773), and if we accept that the sun has existed for nearly 4.6 billion years, we should assume that shadows have an equivalent history. A shadow created by the sun and our body can be regarded as our most familiar avatar. Plato left an interesting trail of thoughts about shadows by Socrates in *Republic*:

Human beings living in an underground den, which has a mouth open towards the light and reaching all along the den; here they have been from their childhood, and have their legs and necks chained so that they cannot move, and can only see before them, being prevented by the chains from turning round their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way, like the screen which marionette players have in front of them, over which they show the puppets [...] and they see only their own shadows, or the shadows of one another, which the fire throws on the opposite wall of the cave [...] To them, I said, the truth would be literally nothing but the shadows of the images. (Plato 2017, 2–3)

Plato’s allegory of the cave has captured the attention of many researchers. Thomas Metzinger uses this metaphor in the context of arguing that the self is like a shadow reflected in a cave (Metzinger 2009). According to Richard Gregory, visual perception is like picking up a shadow, and our eyes are like the cave (Gregory 1997). Metzinger and Gregory argue that “the self” or “perception through the visual system” is a virtual product, where the shadow represents the virtuality. However, the shadow actually affects our visual system beyond the dimension of representation. For example, even in a neutral form with respect to depth, the appearance of depth is reversed when the lighting angle is changed from the top to the bottom (see Fig. 4). This is considered as an illusion caused by the fact that sunlight is emitted from above and the light emitted from below basically did not exist in nature.

The connection between vision and shadow has been suggested, but how can shadow be connected to mirror image? Let us reflect on Feynman’s statement, mentioned at the beginning of [section 2.1](#). According to Feynman: “We cannot imagine ourselves ‘squashed’ back to front”. Now, let us imagine a situation where the light source is behind you and a shadow is cast in front of you. This kind of situation is often experienced in daily life, and is not unique in any way. In this case, the silhouette of the back of the body should be projected on the wall or ground in front of you, but is this really an accurate explanation? Rather, is the silhouette more like the back and front overlapped

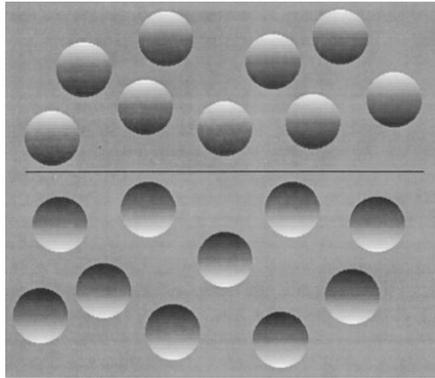


Figure 4. The depth in these displays is conveyed exclusively through shading (Kleffner and Ramachandran 1992, 20).

with each other? If it is true, in a sense we are virtually seeing “ourselves ‘squashed’ back to front.” Let us check the situation in more detail. In the above case, if we raise our right hand, we watch our shadow raise the hand on the right side. When we raise our left hand, our shadow raises the hand on the left side. Therefore, it means that this type of shadow also has plane symmetry – more precisely, “quasi plane symmetry” – and depending on the choice of frame of reference, the shadow would raise its right hand or its left hand. However, we rarely apply the intrinsic frame of reference to shadows. It is not possible to discuss the reasons for that in detail in this paper, but it would be disadvantageous for animal survival to apply the common frame of reference to a shadow immediately while there is a possibility that it is a sign of a predator or enemy. Hence, it is surprising that we tend to apply the common frame of reference to our shadows, even though it might be disadvantageous to our survival. Perhaps we do not have an innate preference for a certain frame of reference, but rather we learn to apply the common frame of reference to objects that are symmetrical to us, and recognize them as part of ourselves.

4 Conclusion

I conclude that we tend to apply the common frame of reference to shadows, but, in some specific cases, we can also apply the intrinsic frame of reference to them. This is consistent with our tendency towards mirror images.

What can be inferred from this chapter is summarized as the proposition that there is a commonality between shadow and mirror image with respect to the choice of the frame of the reference. If this proposition is true, then we may be able to make sense of an almost idealistic statement such as “the

shadow anchors the mirror image to the self.” However, when mirror image and shadow are connected in both the idealistic and phenomenal dimensions, it is very interesting to us and seems to pose a fundamental question.

There is a phenomenon called the chameleon effect. It indicates our tendency to mimic others unconsciously, and to feel an affinity with others who mimic us, unless we notice them doing it (Chartrand and Bargh 1999). This phenomenon is also known as mirroring. In the above sections, I clarified that we generally make our mirror images and shadows belong to ourselves. Is there then a possibility in mirroring that we recognize others who act like us as a virtual mirror image or a shadow of ourselves? It is a mystery.

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Comment by Shunsuke Mukae

Ever since Socrates, there have been various disciplinary approaches to the phenomenon of mirror reversal. This essay starts with an overview of the results of recent optical and psychological approaches. It then attempts to develop (or revisit) a more speculative stage by connecting it to the discussion of the vision of shadows.

The key to this is the frame of reference and mirroring, both of which are related to cognition. Based on previous research, the author shows that left-right reversal is a phenomenon associated with front-back and up-down reversal, both in terms of subjective and objective frames of reference. Interestingly, in connecting this to the discussion of shadows, he uses the phenomenon of mirroring as an interface. This is the feeling of closeness to others who behave in the same way as oneself, including one's mirror image or shadow.

Ito's argument concludes here, but an extension to digital space would add valuable "depth" to the speculative argument. In the world of video games, we often see ourselves in a different way, as avatars. We see ourselves through the eyes of a third person. The hacking of surveillance cameras in *Watch Dogs* (2014) and the sight-jacking in *Siren: Blood Curse* (2008) do not involve a left-right reversal. In the former, only the avatar is mosaicked, and in the latter, the ability is a cursed power that is only given to those who come close to non-human beings. With this in mind, there would certainly be value in extending and exploring Ito's argument in the digital space.

Comment by Konstantin Freybe

Ito approaches plane symmetry as a mystery that he attempts to discuss from a philosophical perspective. He draws knowledge from multiple disciplines: Fine Arts; Physics; Psychology; and Philosophy. His empirical subject area is quite condensed, since mirrors and shadows are commonly known phenomena. His discussion, however, moves on to approach the concept of self on a theoretical level. He shows convincingly that there are multiple frames of reference at play when confronted with mirror images and shadows, and that the philosophical dimension of this requires more than geometrical optics to grasp it appropriately. While the condensed writing progresses in broad strokes, it provides a diverse pallet of impulses that force readers to reconsider what they are talking about when "reflecting" on issues or "mirroring" actions of others. Providing a fresh take on subject-object relations is perhaps the main achievement of Ito's article. Sustaining the relation between rather basic phenomena and elaborate

philosophical concepts, such as “self,” is challenging. However, his attempt to provide diverse theoretical approaches is, in my opinion, best understood as (re-)opening a conversation rather than a conclusive discussion.

